UK Intermediate Mathematical Challenge

THURSDAY 2ND FEBRUARY 2017

Organised by the **United Kingdom Mathematics Trust**
and supported by

Institute
and Faculty
of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
   Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
   When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
   sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
   even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
   rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
   and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

1. What is the value of $\dfrac{2}{5} + \dfrac{2}{50} + \dfrac{2}{500}$?

   A 0.111 B 0.222 C 0.333 D 0.444 E 0.555

2. Each of the diagrams below shows a circle and four small squares. In each case, the centre of the circle is the point where all four squares meet.

   In one of the diagrams, exactly one third of the circle is shaded. Which one?
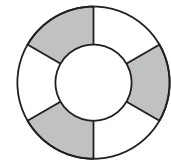
   A B C D E

3. How many squares have 7 as their units digit?

   A 0 B 1 C 2 D 3 E 4

4. Which of the following is *not* the sum of two primes?

   A 5 B 7 C 9 D 11 E 13

5. The diagram shows two circles with the same centre. The radius of the outer circle is twice the radius of the inner circle. The region between the inner circle and the outer circle is divided into six equal segments as shown.

   What fraction of the area of the outer circle is shaded?

   A $\dfrac{3}{7}$ B $\dfrac{3}{8}$ C $\dfrac{3}{9}$ D $\dfrac{3}{10}$ E $\dfrac{3}{11}$

6. The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6.

   What is the difference between the largest angle and the smallest angle?

   A 30° B 40° C 50° D 60° E 70°

7. Four different positive integers are to be chosen so that they have a mean of 2017. What is the smallest possible range of the chosen integers?

   A 2 B 3 C 4 D 5 E 6

8. Which of the following numbers is the largest?

   A 1.3542 B 1.354$\dot{2}$ C 1.35$\dot{4}$$\dot{2}$ D 1.3$\dot{5}$4$\dot{2}$ E 1.$\dot{3}$54$\dot{2}$
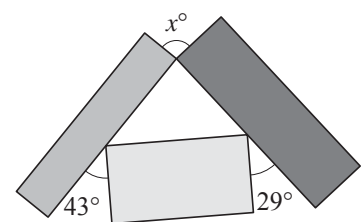
9. The number '*tu*' is the two-digit number with units digit *u* and tens digit *t*. The digits *a* and *b* are distinct, and non-zero. What is the largest possible value of '*ab*' − '*ba*' ?
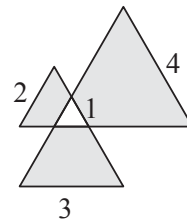
   A 81 B 72 C 63 D 54 E 45

10. The diagram shows three rectangles.
    What is the value of *x*?

    A 108 B 104 C 100 D 96 E 92

11. The diagram shows four equilateral triangles with sides of lengths 1, 2, 3 and 4. The area of the shaded region is equal to n times the area of the unshaded triangle of side-length 1.
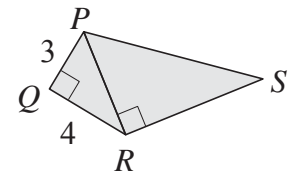
What is the value of n?

A 8   B 11   C 18   D 23   E 26

12. The combined age of Alice and Bob is 39. The combined age of Bob and Clare is 40. The combined age of Clare and Dan is 38. The combined age of Dan and Eve is 44. The total of all five ages is 105.

Which of the five is the youngest?

A Alice   B Bob   C Clare   D Dan   E Eve

13. The diagram shows a quadrilateral $PQRS$ made from two similar right-angled triangles, PQR and PRS. The length of PQ is 3, the length of QR is 4 and $\angle PRQ = \angle PSR$.
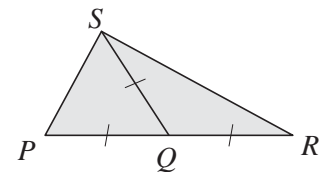
What is the perimeter of $PQRS$?

A 22   B $22\frac{5}{6}$ C 27   D 32   E $45\frac{1}{3}$

14. For what value of x is 64^x equal to 512^5 ?

A 6   B 7.5   C 8   D 16   E 40

15. In the diagram shown, $PQ = SQ = QR$ and $\angle SPQ = 2 \times \angle RSQ$.

What is the size of angle QRS?

A 20°   B 25°   C 30°   D 35°   E 40°

16. The product of two positive integers is equal to twice their sum. This product is also equal to six times the difference between the two integers. What is the sum of these two integers?

A 6   B 7   C 8   D 9   E 10

17. The diagram shows two rectangles and a regular pentagon. One side of each rectangle has been extended to meet at X.

What is the value of x?

A 52   B 54   C 56   D 58   E 60
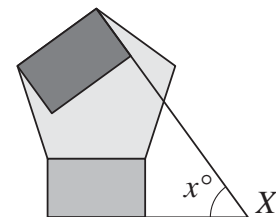
18. A water tank is $\dfrac{5}{6}$ full. When 30 litres of water are removed from the tank, the tank is $\dfrac{4}{5}$ full.

How much water does the tank hold when full?

A 180 litres   B 360 litres   C 540 litres   D 720 litres   E 900 litres

19. $PQRS$ is a square. Point T lies on PQ so that $PT : TQ = 1{:}2$. Point U lies on SR so that $SU : UR = 1 : 2$. The perimeter of $PTUS$ is 40 cm.

What is the area of $PTUS$?

A 40 cm²   B 45 cm²   C 48 cm²   D 60 cm²   E 75 cm²

20. The diagram shows seven circular arcs and a heptagon with equal sides but unequal angles. The sides of the heptagon have length 4. The centre of each arc is a vertex of the heptagon, and the ends of the arc are the midpoints of the two adjacent sides.
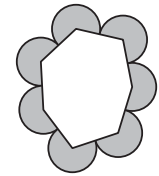
What is the total shaded area?

A 12π B 14π C 16π D 18π E 20π

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21. *Brachycephalus* frogs are tiny − less than 1 cm long − and have three toes on each foot and two fingers on each 'hand', whereas the common frog has five toes on each foot and four fingers on each 'hand'.

Some *Brachycephalus* and common frogs are in a bucket. Each frog has all its fingers and toes. Between them they have 122 toes and 92 fingers.

How many frogs are in the bucket?

A 15   B 17   C 19   D 21   E 23

22. The diagram shows an arc PQ of a circle with centre O and radius 8. Angle QOP is a right angle, the point M is the midpoint of OP and N lies on the arc PQ so that MN is perpendicular to OP.
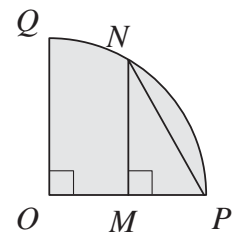
Which of the following is closest to the length of the perimeter of triangle PNM ?

A 17   B 18   C 19   D 20   E 21

23. Two brothers and three sisters form a single line for a photograph. The two boys refuse to stand next to each other.

How many different line-ups are possible?

A 24   B 36   C 60   D 72   E 120

24. The nth term in a certain sequence is calculated by multiplying together all the numbers $\sqrt{1 + \dfrac{1}{k}}$, where k takes all the integer values from 2 to $n + 1$ inclusive. For example, the third term in the sequence is $\sqrt{1 + \dfrac{1}{2}} \times \sqrt{1 + \dfrac{1}{3}} \times \sqrt{1 + \dfrac{1}{4}}$.

Which is the smallest value of n for which the nth term of the sequence is an integer?

A 3   B 5   C 6   D 7   E more than 7

25. The diagram shows a circle with radius 2 and a square with sides of length 2. The centre of the circle lies on the perpendicular bisector of a side of the square, at a distance x from the side, as shown. The shaded region − inside the square but outside the circle − has area 2.
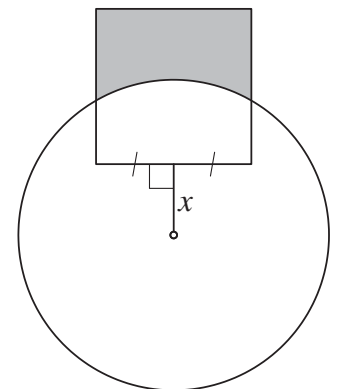
What is the value of x?

A $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{2} - 1$ B $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{4} - 1$ C $\dfrac{\pi}{3} + \dfrac{1}{2}$

D $\dfrac{\pi}{3} + 1$ E $\dfrac{\pi}{3}$